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Einstein–Podolsky–Rosen Correlations and the Preferred Frame

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The correlation function of spin measurements of two spin-$\frac{1}{2}$ particles in two moving inertial frames is derived within the framework of the Lorentz covariant quantum mechanics with the preferred frame. The localization of the particles during the detection and proper transformation properties under the action of the Lorentz group of the spin operator are taken into account. Some special cases and approximations of the calculated correlation function are discussed.

KEY WORDS: Einstein–Podolsky–Rosen *Gedankenexperiment*; quantum correlations; preferred frame; absolute synchronization scheme.

1. INTRODUCTION

Recent analyses of the Einstein–Podolsky–Rosen (EPR) correlations (Bohm, 1951; Einstein *et al.*, 1935) are usually restricted to observers staying in a fixed inertial frame of reference because of very serious difficulties connected with description of EPR-like experiments in frames in a relative motion. The reasons of these difficulties are the incompatibility of the relativity of simultaneity for moving observers with the instantaneous state reduction and the nonexistence of a covariant notion of localization in the relativistic quantum mechanics (Bacry, 1988). The latter deficiency is especially serious because every realistic measurement involves localization in the detector area.

Following some authors (e.g., Bell, 1981), a consistent formulation of quantum mechanics (QM) require a preferred frame (PF) at the fundamental level. A conceptual difficulty related to the notion of PF lies in an apparent contradiction with the Lorentz symmetry. However, Rembieliński (1980, 1997) has shown that it is possible to arrange Lorentz group transformations in such a way that the Lorentz covariance is valid while the relativity principle is broken

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on the quantum level; such an approach is consistent with all the classical phenomena.

The physical meaning of the new form of the Lorentz group transformations lies in an absolute synchronization scheme (ASS) for clocks, which is different from Einstein's scheme (Anderson *et al.*, 1998; Jammer, 1979; Mansouri and Sexl, 1977; Reichenbach, 1969; Will, 1992). Both synchronizations, the new and the standard one, are physically inequivalent on the classical level only for velocities greater than the velocity of light. Furthermore, the causality notion, which is implied by ASS, is more general than the Einstein one and thus it is applicable to nonlocal phenomena.

A Lorentz covariant formulation of QM based on the above mentioned ASS was given by Caban and Rembieliński (1999). In such a formalism it is possible to define Lorentz-covariant localized states and a Lorentz-covariant position operator as well as the spin operator transforming properly under the action of the Lorentz group. Therefore, it allows one to calculate the EPR correlation function for any spin (Rembieliński and Smoliński, 2002), taking into account the localization of the particles during the detection. Note, that the EPR correlations calculated by Czachor (1997a,b) does not comply with these assumptions.

In the rest of this section we briefly describe main features of ASS. The main idea is based on a well-known fact that the definition of time coordinate depends on the procedure used to synchronize clocks (Jammer, 1979; Mansouri and Sexl, 1977). If we restrict ourselves to the time-like or light-like signal propagation, the choice of this procedure is a convention (Anderson *et al.*, 1998; Jammer, 1979; Reichenbach, 1969; Will, 1992). Therefore, the form of Lorentz transformations depends on the synchronization scheme, and we can find a synchronization procedure that leads to such a form of Lorentz transformations that it preserves the instant time hyperplanes.

The transformation of the coordinates between inertial frames $\mathcal{O}_u$ and $\mathcal{O}_{u'}$ reads (Rembieliński, 1980, 1997):

$$x'(u') = D(\Lambda, u)x(u), \tag{1a}$$

$$u' = D(\Lambda, u)u, \tag{1b}$$

where $u = (u^0, \mathbf{u})$ is the four-velocity of PF with respect to $\mathcal{O}_u$, Λ belongs to the Lorentz group, and $D(\Lambda, u)$ is a 4×4 matrix. The Lorentz transformations for rotations are standard while for boosts they take the following form (Rembieliński, 1997):

$$x'^0 = \frac{x^0}{w^0}, \tag{2a}$$

$$\mathbf{x}' = -x^0\mathbf{w} + \mathbf{x} + \frac{\mathbf{w}\cdot\mathbf{x}}{1+\sqrt{1+|\mathbf{w}^2|}}\mathbf{w} - u^0(\mathbf{u}\cdot\mathbf{x})\mathbf{w}, \tag{2b}$$

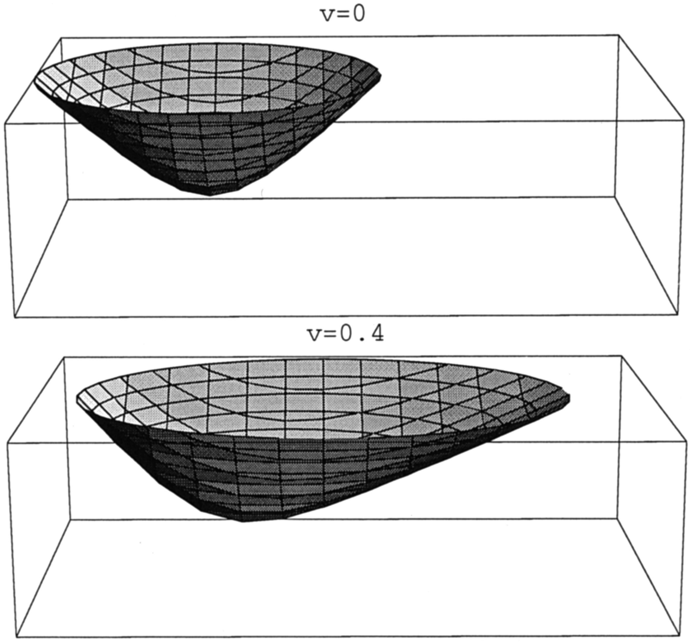

Fig. 1. Four-momentum hyperboloid for a time-like particle as seen by an observer in a different inertial frames in the ASS. This hyperboloid is topologically equivalent to the one in Einstein's synchronization.

where $w = (w^0, \mathbf{w})$ denotes a four-velocity of the frame $\mathcal{O}_{u'}$ as seen by the observer in the frame $\mathcal{O}_u$.

We point out that under Lorentz transformations the time coordinate is only scaled by a positive factor, so the time ordering of events cannot be inverted by any Lorentz transformations, regardless of their space-time separation. The same holds for the 0th component of the four-momentum (see the Figs. 1 and 2). This is important in the QM context because the transformations of time do not involve position operators.

2. LORENTZ COVARIANT QUANTUM MECHANICS, LOCALIZED STATES AND SPIN

The Lorentz covariant QM in the framework of ASS was discussed by Caban and Rembieliński (1999). With each inertial observer in $\mathcal{O}_u$ we associate a Hilbert space $\mathcal{H}_u$, so we have a bundle of Hilbert spaces rather than a single Hilbert space of states. It has been shown (Caban and Rembieliński, 1999) that one can introduce

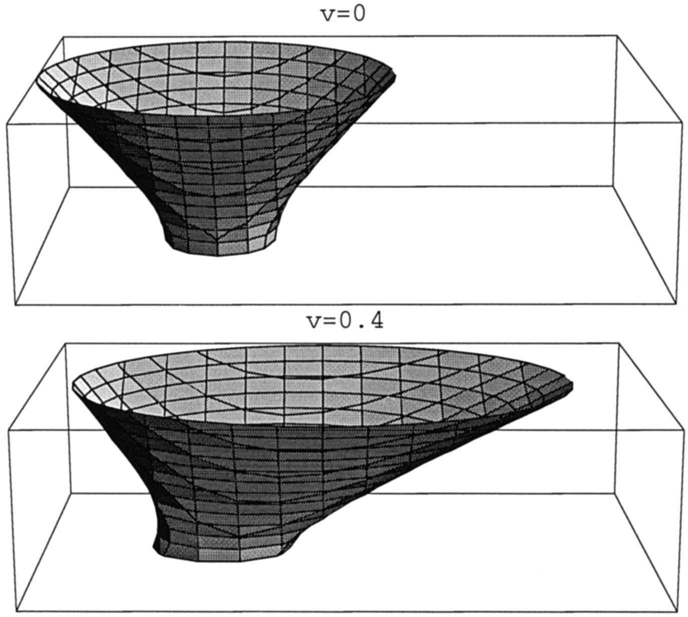

Fig. 2. Four-momentum hyperboloid for a space-like particle as seen by an observer in a different inertial frames in the ASS. Note, that the lower bound of the 0th component of the four-momentum k^0 is the invariant asymptotic boundary given by the equality $k^0 = 0$. Therefore, the situation is completely different than in the standard case, where the sign of k^0 is not Lorentz invariant.

Hermitian momentum and coordinate four-vector operators satisfying

$$[\hat{x}^\mu(u), \hat{p}_\nu(u)] = i\left(\frac{u_\nu \hat{p}^\mu(u)}{u_\lambda \hat{p}^\lambda(u)} - \delta^\mu_\nu\right), \tag{3a}$$

$$[\hat{p}_\mu(u), \hat{p}_\nu(u)] = 0, \tag{3b}$$

$$[\hat{x}^\mu(u), \hat{x}^\nu(u)] = 0. \tag{3c}$$

The time operator $\hat{x}^0$ commutes with all the observables, what allows us to interpret the time as a parameter just like in the standard nonrelativistic QM. Moreover,

$$[\hat{x}^i, \hat{p}_k] = i\delta^i_k, \quad [\hat{x}^i, \hat{p}_0] = \frac{\hat{p}^i}{\hat{p}^0}, \quad [\hat{x}^\mu(u), \hat{p}^2(u)] = 0. \tag{4}$$

The commutation relations (3) are covariant in ASS.

Applying the Wigner method one can easily determine the action of the Lorentz group on the base vectors (Caban and Rembieliński, 1999)

$$U(\Lambda)|k, u, m; s, \sigma\rangle = \mathcal{D}^s(R_{(\Lambda,u)})_{\lambda\sigma}|k', u', m; s, \lambda\rangle, \tag{5}$$

where

$$\hat{p}_\mu(u)|k, u, m; s, \sigma\rangle = k_\mu |k, u, m; s, \sigma\rangle \tag{6}$$

and

$$\langle k, u, m; s, \lambda | k', u, m; s', \lambda'\rangle = 2k^0 \delta^3(\underline{\mathbf{k}}' - \underline{\mathbf{k}})\delta_{s's}\delta_{\lambda'\lambda} \tag{7}$$

(hereafter $\underline{\mathbf{k}}$ denotes the vector formed from covariant components of the momentum, i.e., $\underline{\mathbf{k}} = (k_1, k_2, k_3)$).

Now we can construct the localized states (i.e., the eigenvectors of the position operator) and the covariant spin operator. Eigenstates of the position operator $\hat{\mathbf{x}}(u)$ (locked up in the $t_0 = 0$) are of the form

$$|\mathbf{x}, u, m; s, \sigma\rangle = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3\underline{\mathbf{k}})}{2\omega(\underline{\mathbf{k}})} \sqrt{u^\nu k_\nu}\, e^{i\underline{\mathbf{k}}\cdot\mathbf{x}} |k, u, m; s, \sigma\rangle, \tag{8}$$

where $\omega(\underline{\mathbf{k}}) = k^0$ is a positive solution of the dispersion relation.

The spin operator commute with the position and the following relations hold:

$$\hat{\mathbf{S}}(u)|\mathbf{x}, u, m; s, \tau\rangle = \Sigma^s_{\sigma\tau} |\mathbf{x}, u, m; s, \sigma\rangle, \tag{9}$$

where Σ^s are the standard generators of rotation in $(2s + 1)$-dimensional representation, and

$$U(\Lambda)\hat{S}^i(u)U^\dagger(\Lambda) = R^{T\;\;i}_{(\Lambda,u)\,j}\hat{S}^j(u'), \tag{10}$$

where $R_{(\Lambda,u)}$ is a Wigner rotation. Moreover, $\hat{S}^i(u)$ fulfill the standard commutation relations

$$[\hat{S}^i(u), \hat{S}^j(u)] = i\epsilon^{ijk}\hat{S}^k(u). \tag{11}$$

Let $\mathbf{n}$ be a unit vector. Because $[\hat{\mathbf{S}}(u), \hat{\mathbf{x}}(u)] = 0$ we can introduce a set of common eigenvectors of $\hat{\mathbf{x}}(u)$ and $\mathbf{n}\cdot\hat{\mathbf{S}}(u)$; namely

$$\hat{\mathbf{x}}(u)|\mathbf{x}, \mathbf{n}, u, m; s, \lambda\rangle = \mathbf{x}|\mathbf{x}, \mathbf{n}, u, m; s, \lambda\rangle, \tag{12a}$$

$$\mathbf{n}\cdot\hat{\mathbf{S}}(u)|\mathbf{x}, \mathbf{n}, u, m; s, \lambda\rangle = \lambda|\mathbf{x}, \mathbf{n}, u, m; s, \lambda\rangle. \tag{12b}$$

Thus we can construct a projector onto a region Ω and onto the spin component in the $\mathbf{n}$ direction in the frame $\mathcal{O}_u$:

$$P^\lambda_{\Omega,\mathbf{n}}(u) = \int_\Omega d^3\mathbf{x}|\mathbf{x}, \mathbf{n}, u, m; s, \lambda\rangle\langle\mathbf{x}, \mathbf{n}, u, m; s, \lambda|, \tag{13}$$

which transforms under Lorentz group transformations as follows

$$U(\Lambda)P^\lambda_{\Omega,\mathbf{n}}(u)U^\dagger(\Lambda) = P^\lambda_{\Omega',n'}(u'). \tag{14}$$

In the above formula $\mathbf{n}' = R_{(\Lambda,u)}\mathbf{n}$ and the region Ω' is obtained from the region Ω by the Lorentz transformation $x'^k = D(\Lambda, u)^k{}_i x^i$. There is no analog of formula (14) in standard relativistic QM.

3. EINSTEIN–PODOLSKY–ROSEN CORRELATIONS

In this section we employ the formalism introduced above to calculation of the correlation function of the EPR-type experiment.

We consider distinguishable particles, say α and β; vectors describing pure states belong to the tensor product

$$\mathcal{H}_{\alpha}^{s_\alpha}(u) \otimes \mathcal{H}_{\beta}^{s_\beta}(u). \tag{15}$$

If we assume that the observer $\mathcal{A}$ registers the particle α and the observer $\mathcal{B}$ registers the particle β, then the corresponding observables read

$$M_{A,\boldsymbol{a}}(u_A) = \sum_{\mu_\alpha=-s_\alpha}^{s_\alpha} \mu_\alpha P_{A,\boldsymbol{a}}^{\mu_\alpha}(u_A) \otimes I \equiv \sum_{\mu_\alpha=-s_\alpha}^{s_\alpha} \mu_\alpha \Pi_{A,a}^{\mu_\alpha}, \tag{16a}$$

$$M_{B,\boldsymbol{b}}(u_B) = I \otimes \sum_{\mu_\beta=-s_\beta}^{s_\beta} \mu_\beta P_{B,\boldsymbol{b}}^{\mu_\beta}(u_B) \equiv \sum_{\mu_\beta=-s_\beta}^{s_\beta} \mu_\beta \Pi_{B,\boldsymbol{b}}^{\mu_\beta}. \tag{16b}$$

Let us describe events during an EPR experiment. Firstly, assume that the initial state is described by the density matrix $\rho(u_A, t_A)$. Now, i.e., at the time t_A, the observer $\mathcal{A}$ measures $M_{A,\boldsymbol{a}}(u_A)$ with selection of μ_a. For the observer $\mathcal{B}$ this time is $t_B = D(\Lambda, u_A)^0{}_0 t_A$ and then he measures $M_{B,\boldsymbol{b}}(u_B)$ with selection of μ_b.

The probability $p(\mu_a)$ that the observer $\mathcal{A}$ has measured value μ_a and the probability $p(\mu_b|\mu_a)$ that the observer $\mathcal{B}$ has measured the value μ_b if the observer $\mathcal{A}$ had measured μ_a are

$$p(\mu_a) = \mathrm{Tr}\left[\rho(u_A, t_A)\Pi_{A,\boldsymbol{a}}^{\mu_a}\right], \tag{17a}$$

$$p(\mu_b|\mu_a) = \frac{1}{p(\mu_a)}\mathrm{Tr}\left[\rho(u_A, t_A)\Pi_{A,\boldsymbol{a}}^{\mu_a} U^\dagger(\Lambda)\Pi_{B,\boldsymbol{b}}^{\mu_b} U(\Lambda)\Pi_{A,\boldsymbol{a}}^{\mu_a}\right] \tag{17b}$$

(recall that in $\mathcal{H}_\alpha \otimes \mathcal{H}_\beta$, $U(\Lambda) = U(\Lambda)_\alpha \otimes U(\Lambda)_\beta$). The correlation function then reads

$$C(\mathbf{a}, \mathbf{b}) = \mathrm{Tr}[\rho(u_A, t_A) M_{A,\boldsymbol{a}} U^\dagger(\Lambda) M_{B,\boldsymbol{b}} U(\Lambda)]. \tag{18}$$

Let us assume that the initial state is a singlet state of spin-$\frac{1}{2}$ particles (i.e., $s_\alpha = s_\beta = \frac{1}{2}$), so it can be written in the position basis as

$$|\Psi\rangle = \sum_{\lambda_\alpha,\lambda_\beta=-\frac{1}{2}}^{\frac{1}{2}} \int d^3\mathbf{x} \int d^3y\, 2u^0 \psi_{\lambda_\alpha\lambda_\beta}(\mathbf{x},\mathbf{y},u_A) \times \left|\mathbf{x}, u_A, m_\alpha; \frac{1}{2}, \lambda_\alpha\right\rangle \otimes \left|\mathbf{y}, u_A, m_\beta; \frac{1}{2}, \lambda_\beta\right\rangle, \quad (19)$$

where

$$[\psi_{\lambda_\alpha\lambda_\beta}](\mathbf{x},\mathbf{y},u_A) = \frac{i}{\sqrt{2}}\chi(\mathbf{x},\mathbf{y},u_A)\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}. \quad (20)$$

Therefore

$$C(\mathbf{a},\mathbf{b}) = -\frac{1}{4}\int_A d^3\mathbf{x} \int_{B_{\mathcal{A}}} d^3\mathbf{y} |\chi(\mathbf{x},\mathbf{y},u_A)|^2 (\mathbf{a}\cdot R^T_{(\Lambda,u_A)}\mathbf{b}), \quad (21)$$

i.e., up to a factor

$$C(\mathbf{a},\mathbf{b}) \propto \mathbf{a}\cdot R^T_{\Lambda,u_A}\mathbf{b}. \quad (22)$$

Now, the only thing we need is to calculate the explicit form of the Wigner rotation matrix in ASS. Unfortunately, the resulting formula is rather complicated, so instead discussing the general case, we consider a number of special ones.

1. If both the measurements are performed in the same inertial frame or if one of the observers performs his measurement in PF, we find that

$$C(\mathbf{a},\mathbf{b}) \propto \mathbf{a}\cdot\mathbf{b} = \cos\theta_{\mathbf{ab}.} \quad (23)$$

2. If the velocities of PF are high, we obtain the approximated formula of the form

$$C(\mathbf{a},\mathbf{b}) \propto \mathbf{a}\cdot\mathbf{b} - \frac{1}{1+\mathbf{n}_A\cdot\mathbf{n}_B}[(\mathbf{a}\cdot\mathbf{n}_A)(\mathbf{b}\cdot\mathbf{n}_A) + (\mathbf{a}\cdot\mathbf{n}_B)(\mathbf{b}\cdot\mathbf{n}_B) + (\mathbf{a}\cdot\mathbf{n}_B)(\mathbf{b}\cdot\mathbf{n}_A) - (1+2\mathbf{n}_A\cdot\mathbf{n}_B)(\mathbf{a}\cdot\mathbf{n}_A)(\mathbf{b}\cdot\mathbf{n}_B)], \quad (24)$$

   where $\mathbf{n}_A$ and $\mathbf{n}_B$ denote the directions of the velocities of PF with respect to the observers.
3. If the velocities of PF are small, i.e. $|\boldsymbol{\sigma}_A| \ll 1$ and $|\boldsymbol{\sigma}_B| \ll 1$ we can approximate the correlation function by

$$C(\mathbf{a},\mathbf{b}) \propto \mathbf{a}\cdot\mathbf{b} + \frac{(\mathbf{a}\times\mathbf{b})\cdot(\boldsymbol{\sigma}_A\times\boldsymbol{\sigma}_B)}{2}. \quad (25)$$

   This last case may correspond to the correlation experiments performed on the Earth and identification of PF with cosmic background radiation frame.

4. CONCLUSIONS

We have shown that in the framework of the Lorentz covariant QM with PF one can build the formalism allowing the calculation of the correlation function in the EPR-type experiments performed in moving inertial frames. Because the resulting formula for the correlation function depends on velocities of the PF it can be helpful with the proposal of a realistic experiment that can answer the question of the existence of quantum mechanical preferred frame. It is important to stress that the dependence of the EPR correlation function on PF velocity cannot be removed by expressing the correlation function by classical velocities given in the Einstein's synchronization scheme. This means that the Lorentz covariant quantum mechanics must distinguish a preferred frame.

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